Comments on "Refraction at a Curved Dielectric Interface: Geometrical Optics Solution"

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In the above paper[1], Lee *et al.* consider the refraction of waves at a curved dielectric interface between media of uniform refractive index. They quote an earlier paper by us [2], and infer that our results are either inadequate or incorrect. We should like to have the opportunity to state that we have no reason to doubt the validity of our results, which in any case were independently verified by Jones [3], and to point out how the erroneous conclusions of Lee *et al.* arise.

The two papers, [1] and [2], are addressed to different problems. Lee *et al.* determine the fields *at finite distances from the interface*, i.e., at positions 2 and 3 in [1, fig. 1], whereas our paper considers only reflection *at the interface*, i.e., at position 1 in [1, fig. 1]. In other words, our calculation determines generalizations of the transmission and reflection coefficients T and R of (2.4) for both refracting and tunnelling rays, and therefore the "divergence factors" do not enter our analysis. The fields expressed by [1, (4.11)] are used to calculate T and R for refracting rays, and should not be confused with those of (4.8) which apply at finite distances from the interface.

This misinterpretation of our results also accounts for the incorrect statements made below [1, (4.12)]. Our power transmission coefficient depends only on the fields at the interface and is therefore independent of the "divergence factors".

Finally, the assertion that the phase factor $\exp(-jk_2y)$ is missing from [1, 4.11a)] is incorrect. To demonstrate this, we refer to [2], and deduce from (29a), (30a), and (31), together with continuity of E_z at $y=0$, that

$$cAi\,(\Delta \exp[2\pi i/3]) = 2a\psi/(\psi+1) \qquad (1)$$

and consequently the y-dependence of the transmitted field is expressible as

$$\frac{c}{a}Ai\,(\xi \exp[2\pi i/3]) = \frac{2\psi}{\psi+1}\,\frac{Ai(\xi \exp[2\pi i/3])}{Ai(\Delta \exp[2\pi i/3])}. \qquad (2)$$

It is clear from (28b) and (34) that $\xi, \Delta \to -\infty$ as $\alpha_t \to 0$. With the help of (A5) and its first derivative, we find from (32) and (33b) that as $\alpha_t \to 0$

$$\psi \cong -(i/\gamma)/\left[-i(-\Delta)^{1/2}\right] = 1/\sin\alpha_c = n_1/n_2. \qquad (3)$$

Substituting (3) into (2) and applying (A5) in the same limit

$$\frac{c}{a}Ai\,(\xi \exp[2\pi i/3]) \cong \frac{2n_1}{n_1+n_2}\exp\left(\frac{2i}{3}\left[(-\xi)^{3/2} - (-\Delta)^{3/2}\right]\right). \qquad (4)$$

We then Taylor expand to obtain in the limit $\alpha_t \ll \alpha_c$

$$(-\xi)^{3/2} \cong \frac{k_1\rho}{2}\,\frac{\sin^3\alpha_c}{\sin^2\alpha_t}\left(1 + 3\frac{y}{\rho}\,\frac{\sin^2\alpha_t}{\sin^2\alpha_c}\right) = (-\Delta)^{3/2} + \frac{3}{2}k_2 y \qquad (5)$$

since $\sin\alpha_c = n_2/n_1$. Substituting (5) and (4) into (30a) we obtain the factor $\exp(-ik_2y)$. In the notation of [1], this is the phase factor allegedly missing from (4.11a).

We are grateful to Dr. R. A. Sammut for bring the disagreement in [1] to our attention.

Reply[2] by S. W. Lee, M. S. Sheshadri, V. Jamnejad, and R. Mittra[3]

The fact that the divergence factors are missing in (29) and (30) of [2] is now being explained by Love and Synder that those expressions are valid only at the interface ($y=0$), and not valid elsewhere. However, we do not see such a qualification in their paper [2]. Hence, we stand by our comment (i) in [1].

We agree with Love and Snyder that [2, (30a)] does contain the phase factor $\exp(-jk_2y)$. Thus, our comment (ii) in [1] is not justified.

REFERENCES

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Manuscript received January 5, 1983.

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